

*Advanced course on*

HIGH RESOLUTION ELECTRONIC MEASUREMENTS  
IN NANO-BIO SCIENCE

POLITECNICO DI MILANO



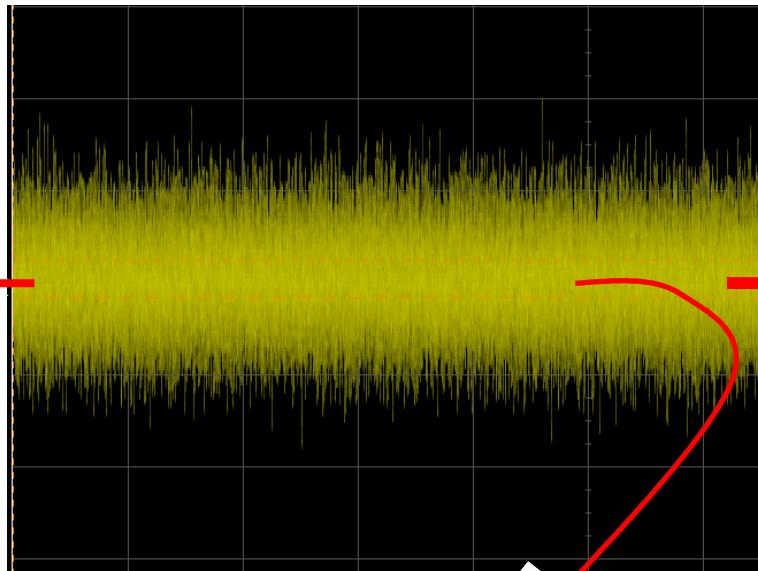
**IT IS ALL ABOUT NOISE :**  
*a practical review*



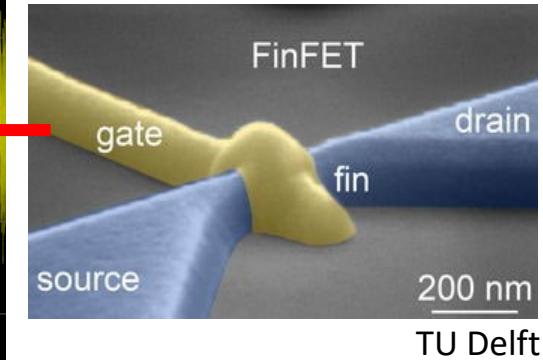
**Marco Sampietro**

# Noise in Instruments, Circuits & Devices

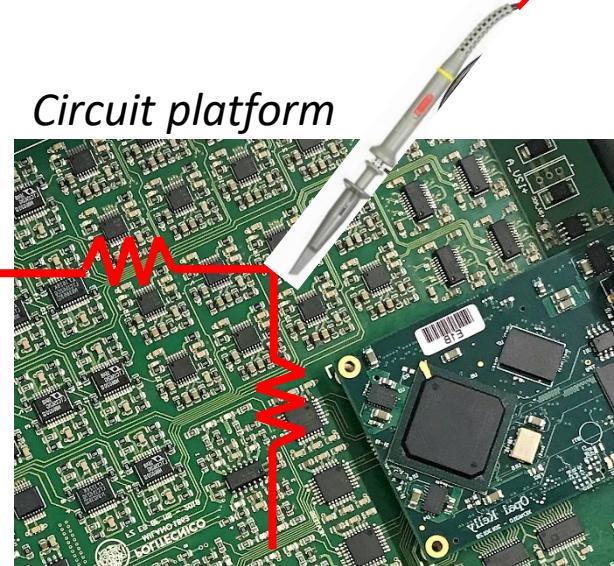
*Signal generator*



*NanoDevices*



*Circuit platform*



# OUTLINE of the lesson

## PRINCIPLES on noise

Noise characteristics

Bandwidth vs Signal response vs Noise

30 min

Calculation of RMS in presence of 1/f noise

## SOURCES of noise

Resistors and capacitors

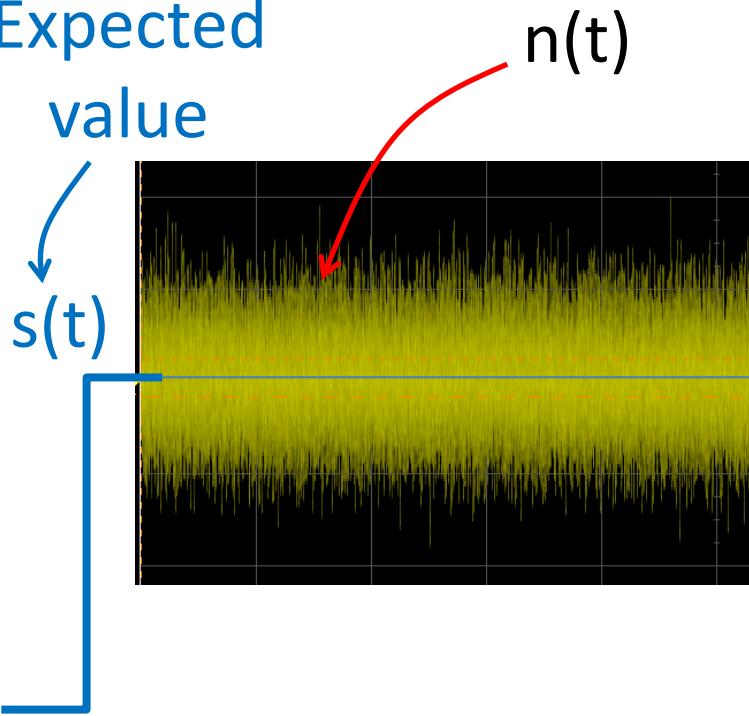
Dielectric noise spectrum and RMS

15 min

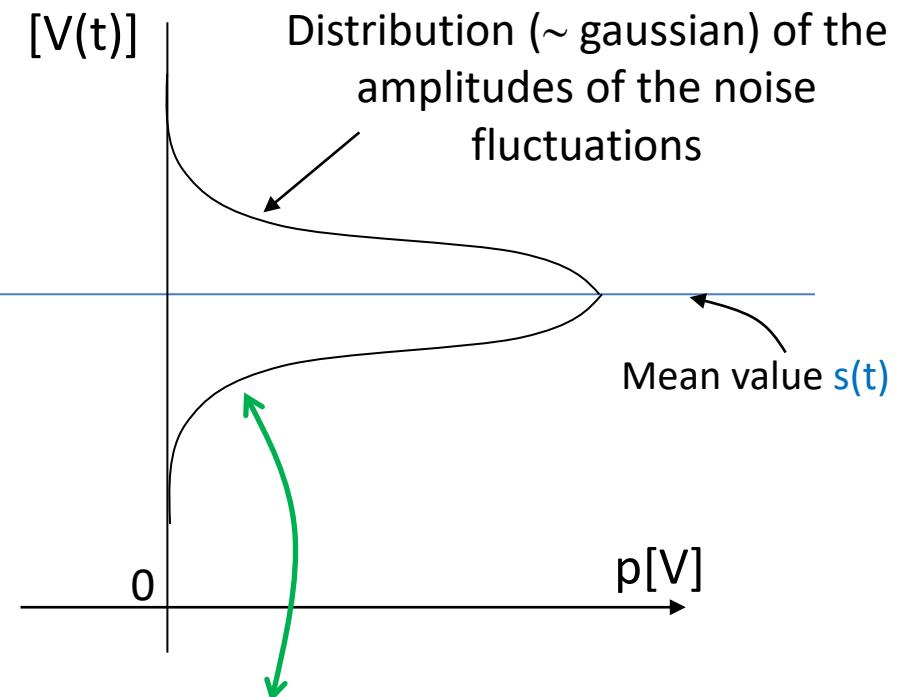
# NOISE CHARACTERISTICS

## - Mean Value -

Expected  
value



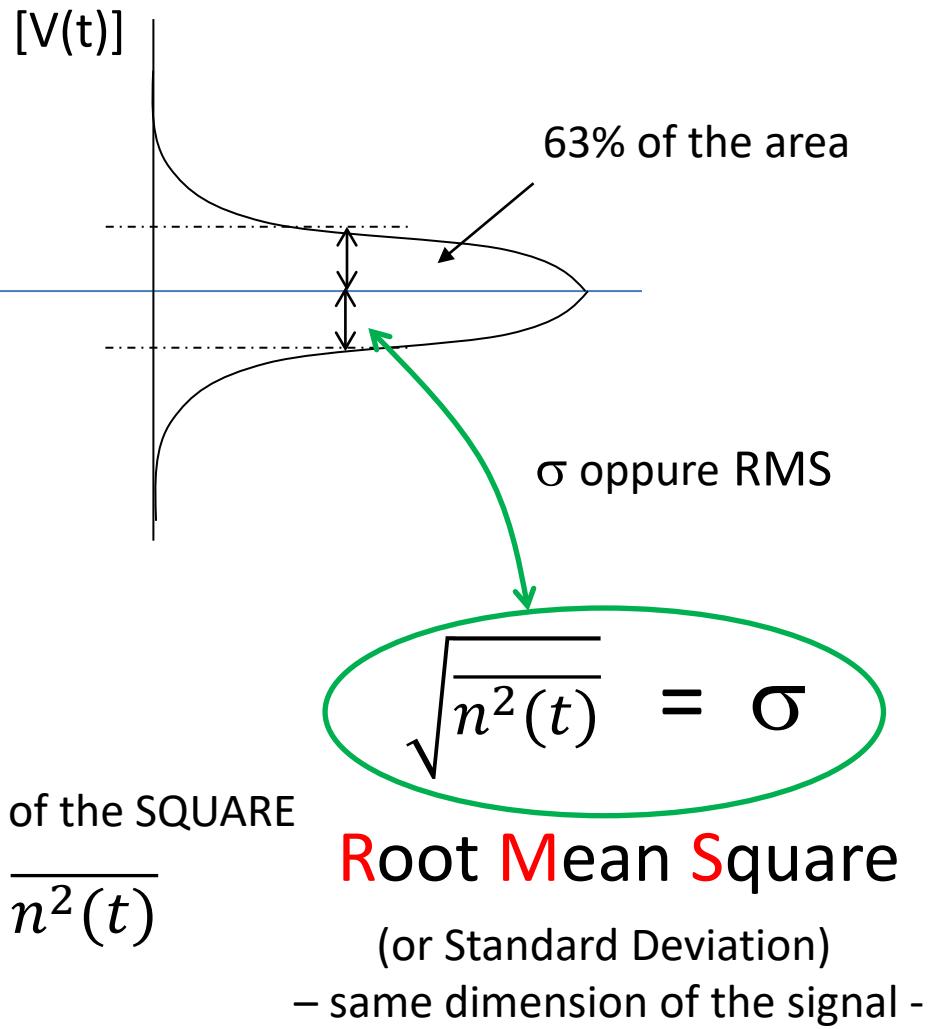
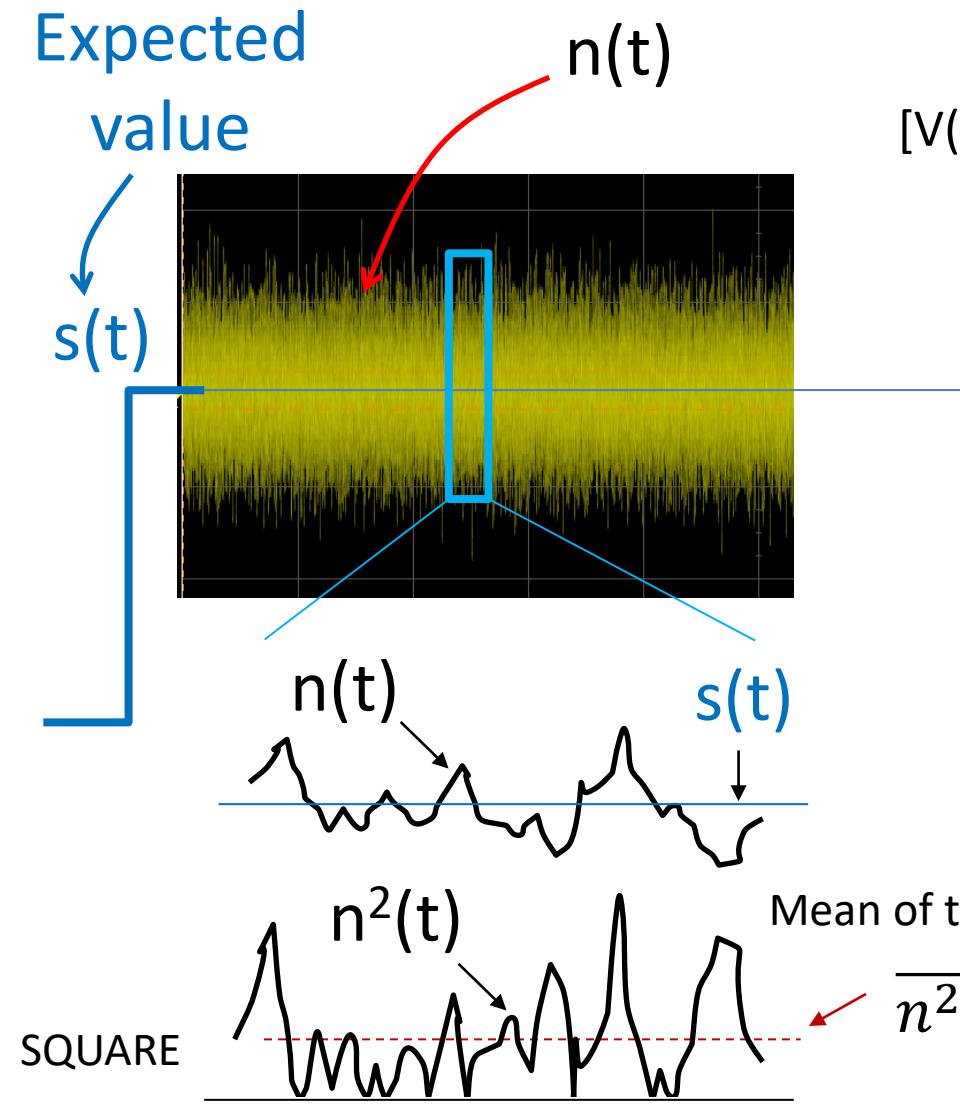
[ $V(t)$ ]



Noise average value  $\overline{n(t)}=0$

# NOISE CHARACTERISTICS

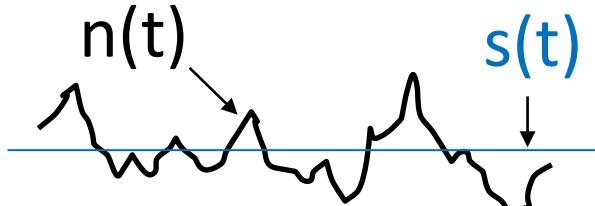
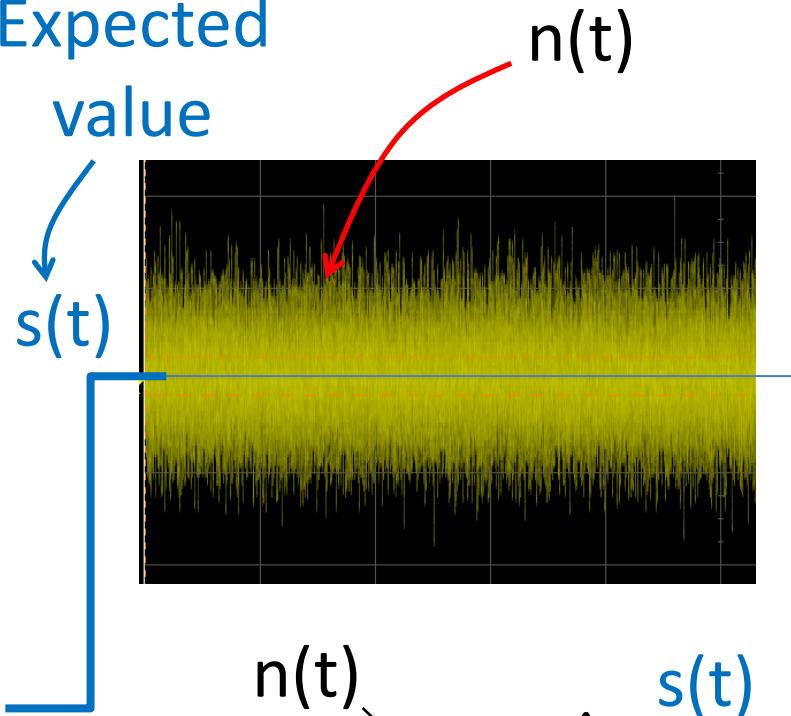
## - Standard deviation -



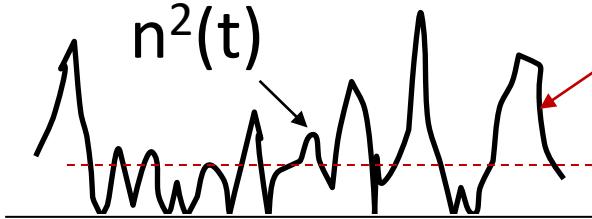
# NOISE CHARACTERISTICS

## - Noise Power -

Expected  
value



$[V]^2$  or  $[I]^2$  : **POWER** of the Noise



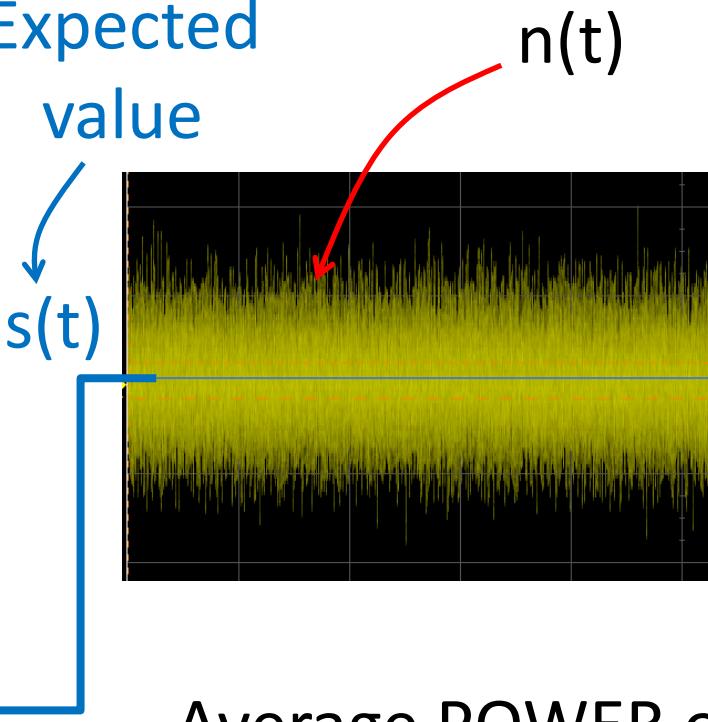
SQUARE  
MEAN power of the Noise

$$\overline{n^2(t)} = \sigma^2$$

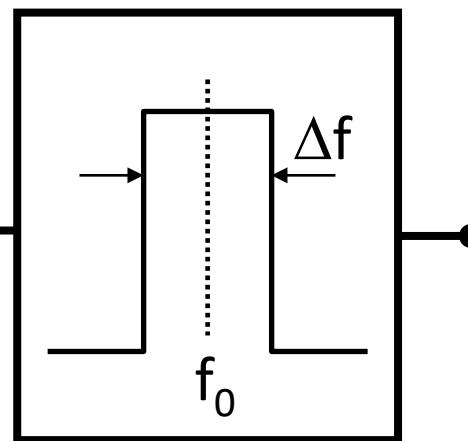
# NOISE CHARACTERISTICS

## - *Power spectral density* -

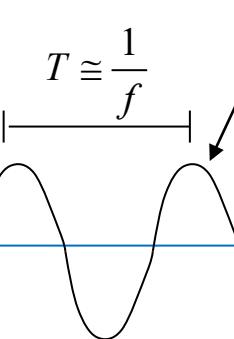
Expected  
value



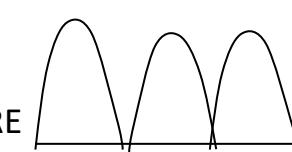
Bandpass filter



Amplitude & phase  
of noise components  
varies casually  
with time



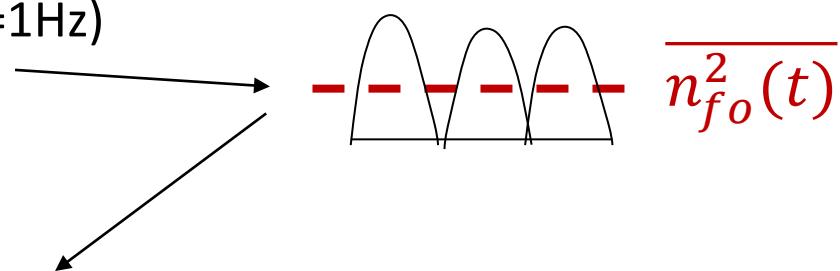
$$n_{fo}(t)$$



$$n_{fo}^2(t)$$

Average POWER of each ( $\Delta f=1\text{Hz}$ )  
single frequency of noise

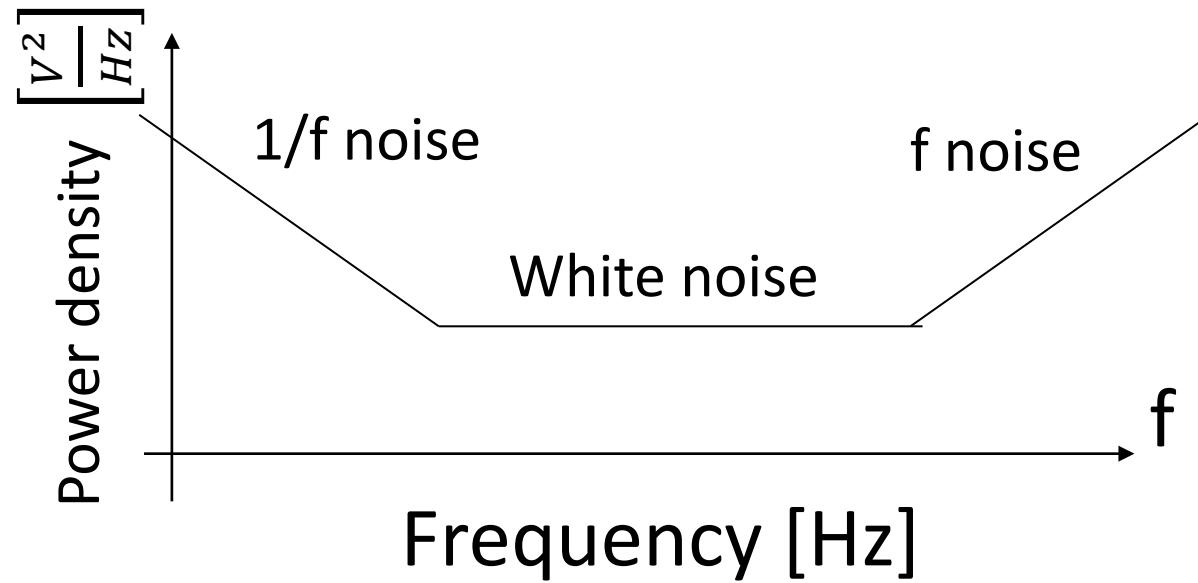
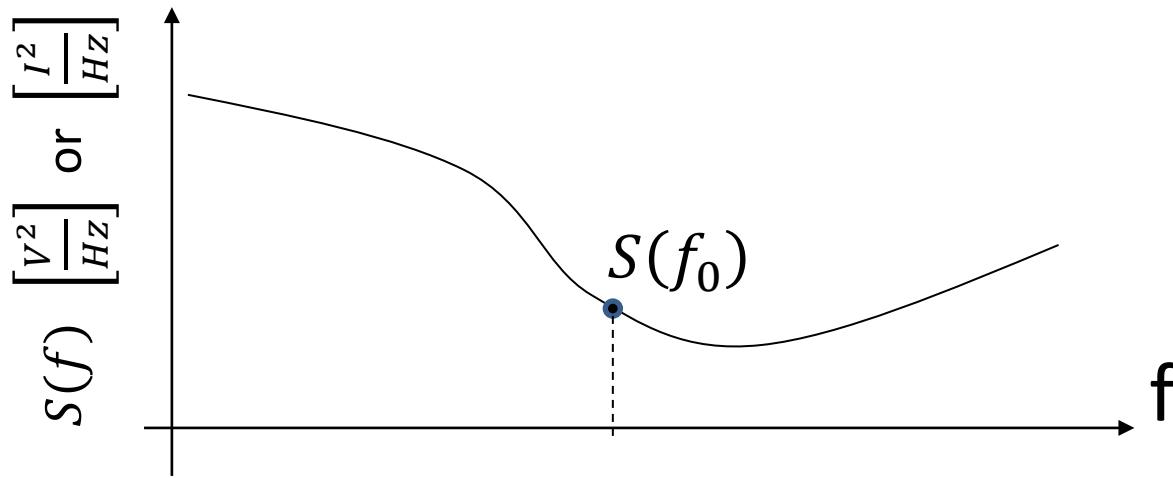
that is  
↓



$$\text{POWER SPECTRAL DENSITY : } S(f_0) \equiv \overline{n_{fo}^2(t)} \left[ \frac{V^2}{\text{Hz}} \right] \text{ or } \left[ \frac{I^2}{\text{Hz}} \right]$$

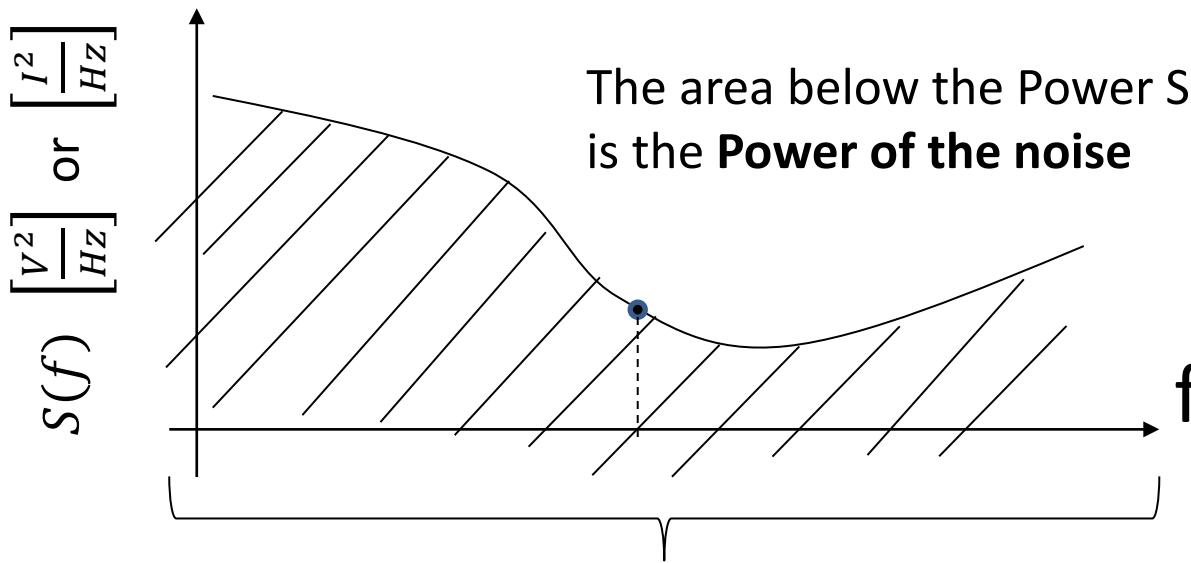
# NOISE CHARACTERISTICS

- *Power spectrum* -



# NOISE CHARACTERISTICS

## - Noise vs Bandwidth -



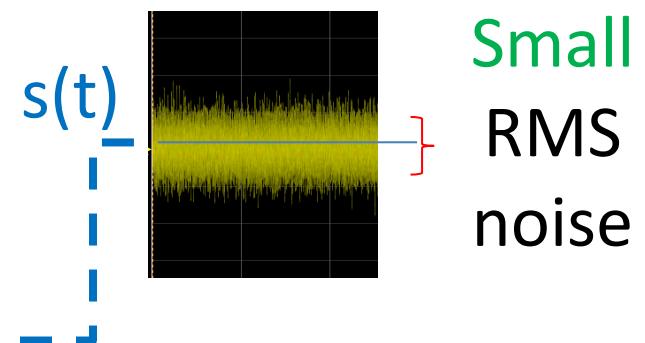
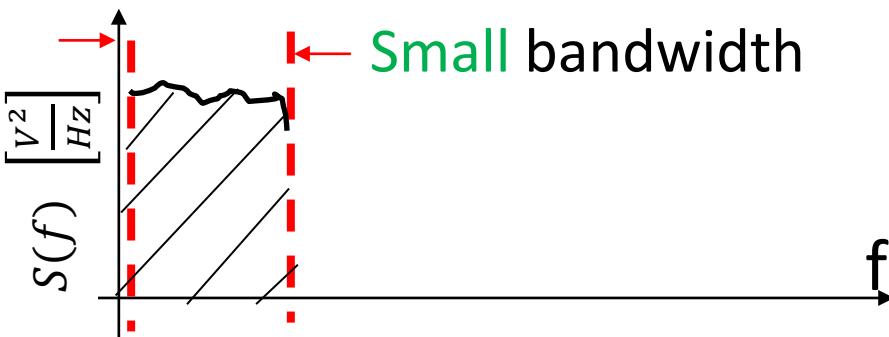
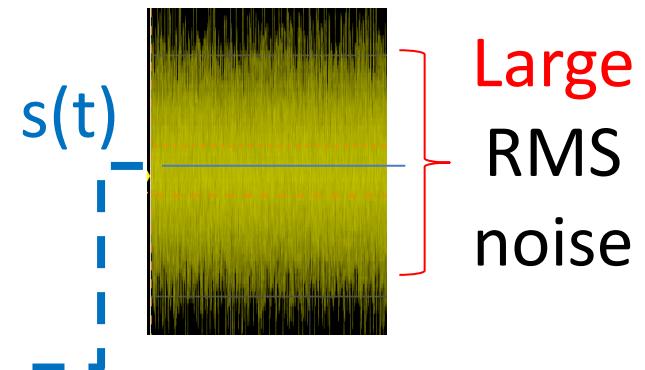
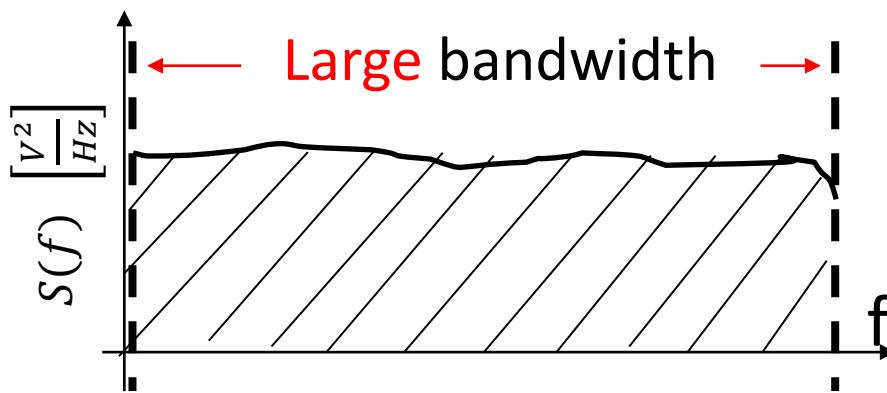
The Power of all  
noise components

$$\int_0^\infty S(f) df = \overline{n^2(t)}$$

The Power of the noise  
as «seen» in time

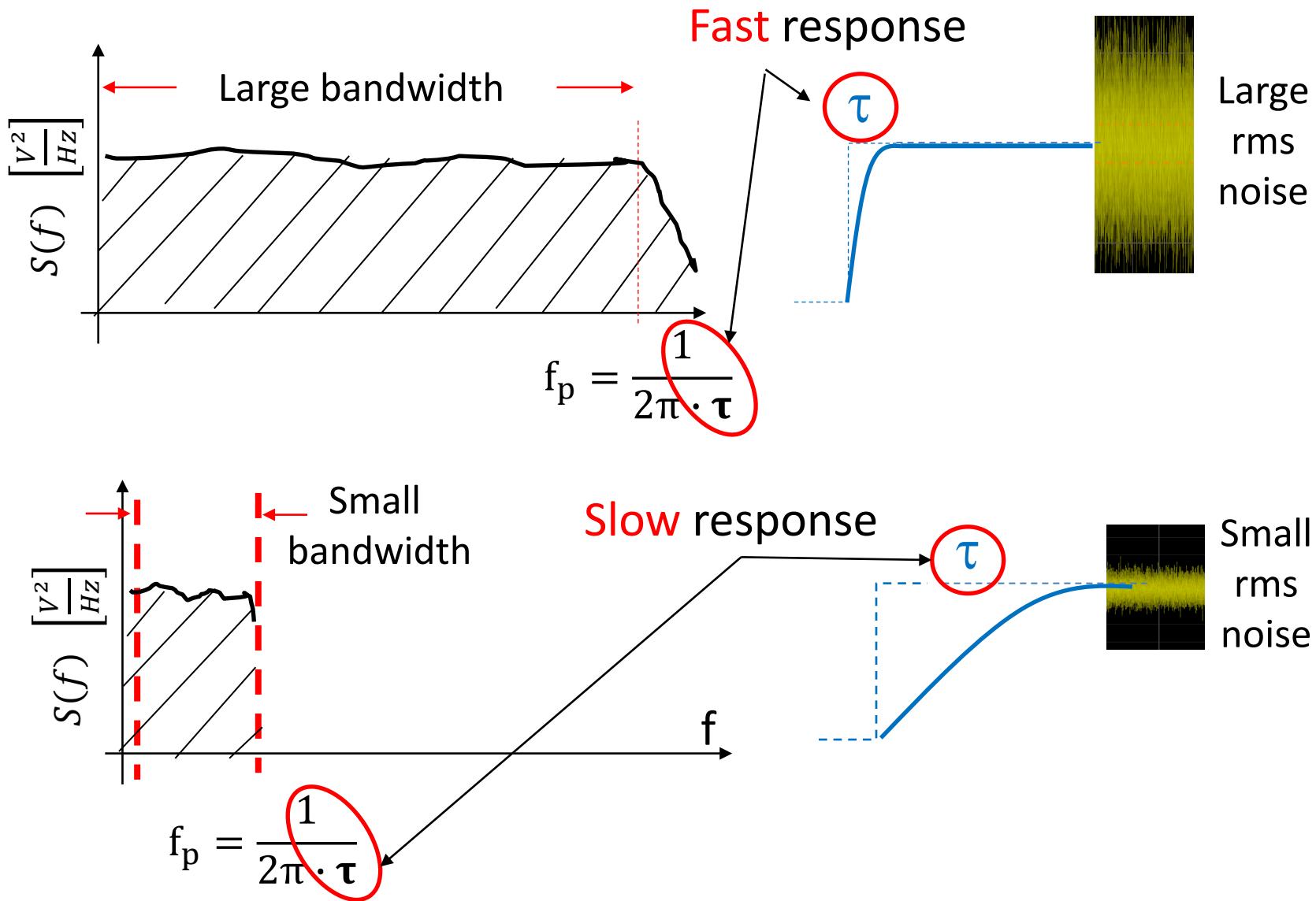
# NOISE vs BANDWIDTH

If white noise :  $\sqrt{S(f) \cdot \text{BW}} = \sqrt{n(t)^2} = \text{RMS}_{\text{noise}}$



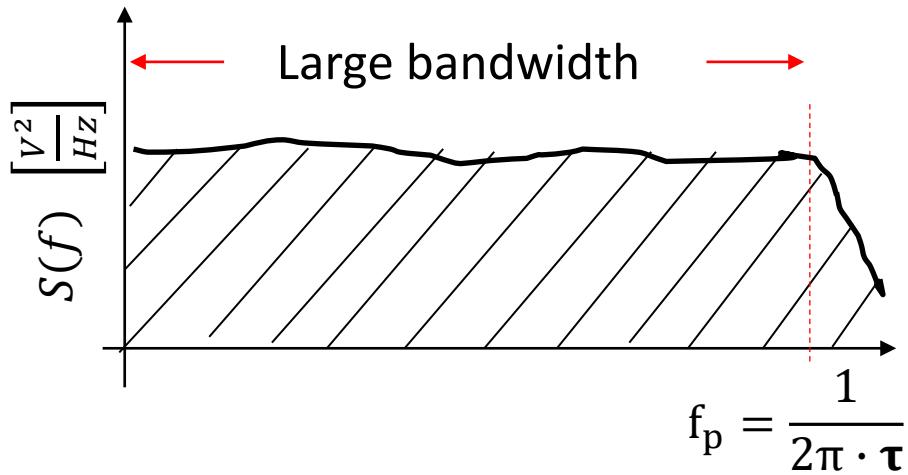
To obtain small noise, reduce the bandwidth as much as possible But ...

# BANDWIDTH vs SIGNAL RESPONSE

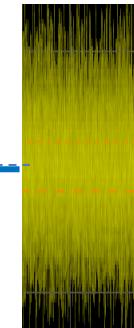
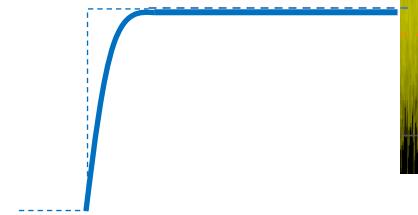


**NEVER USE MORE BANDWIDTH THAN REQUIRED by the SIGNAL**

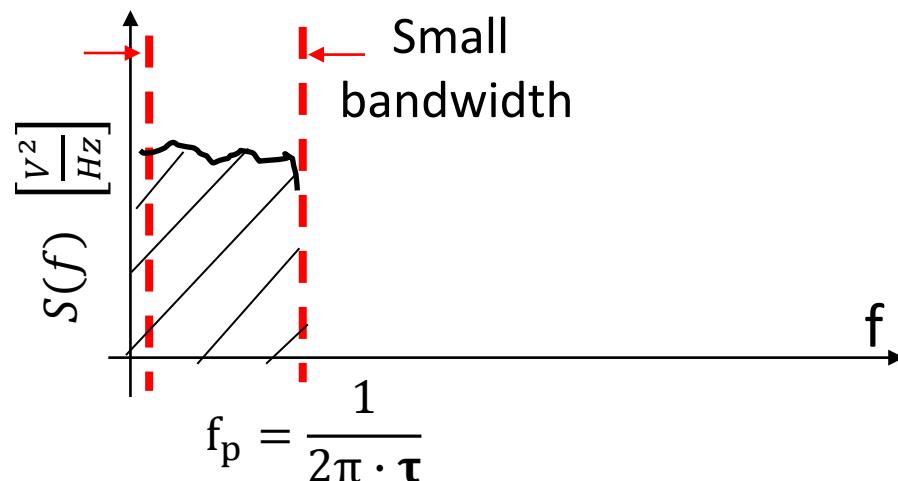
# BANDWIDTH vs Time AVERAGING



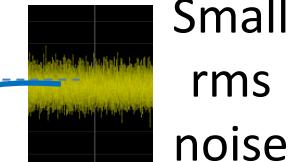
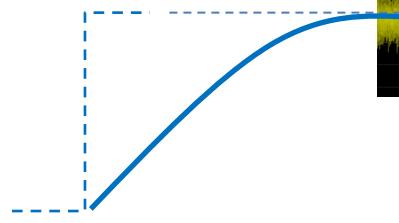
Fast response



Large  
rms  
noise



Slow response



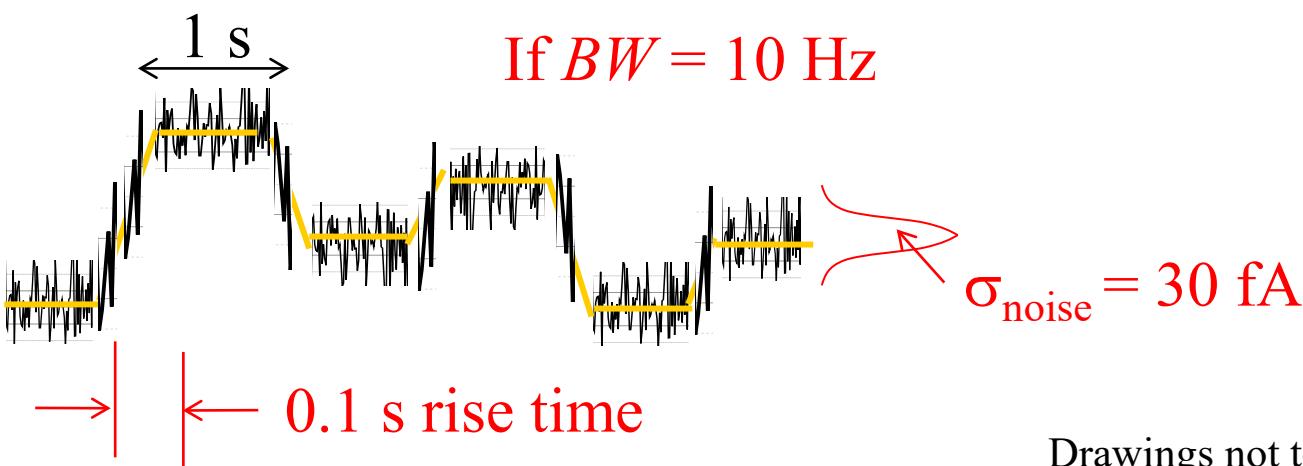
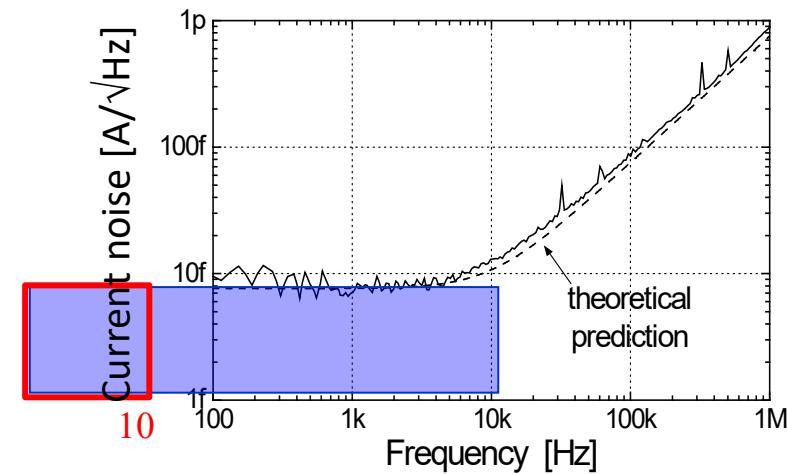
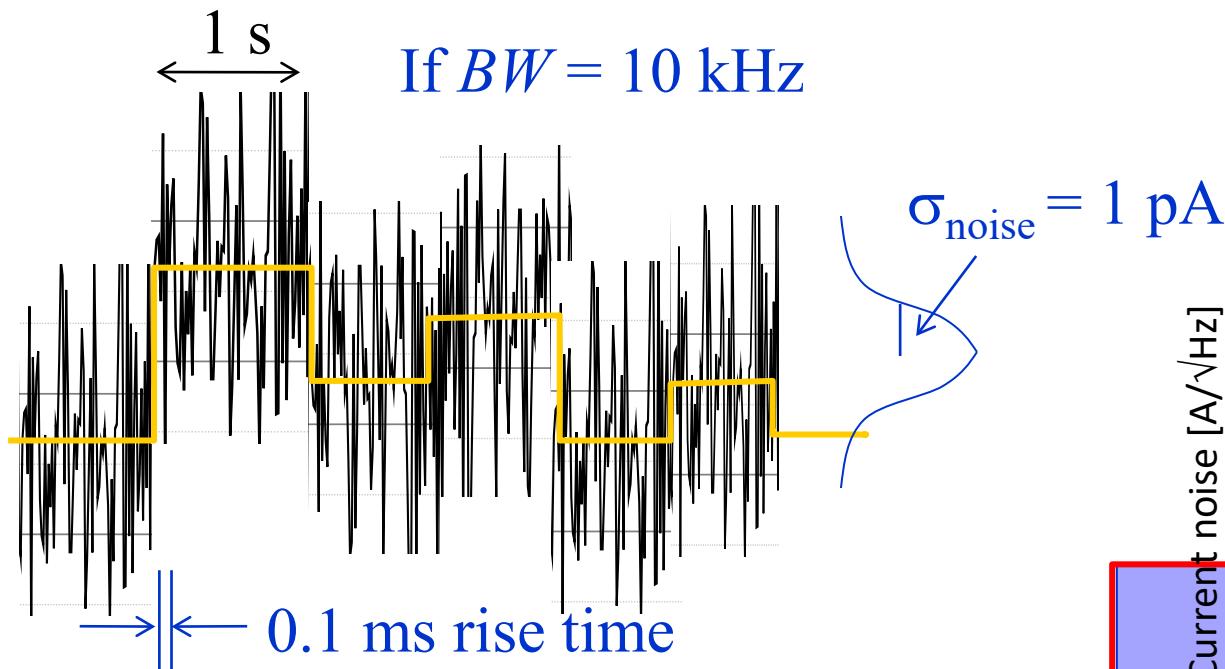
Small  
rms  
noise

Long time for averaging  
the noise fluctuations

Good

**AVERAGE AS LONG AS YOU CAN**

# NOISE vs BANDWIDTH vs SIGNAL

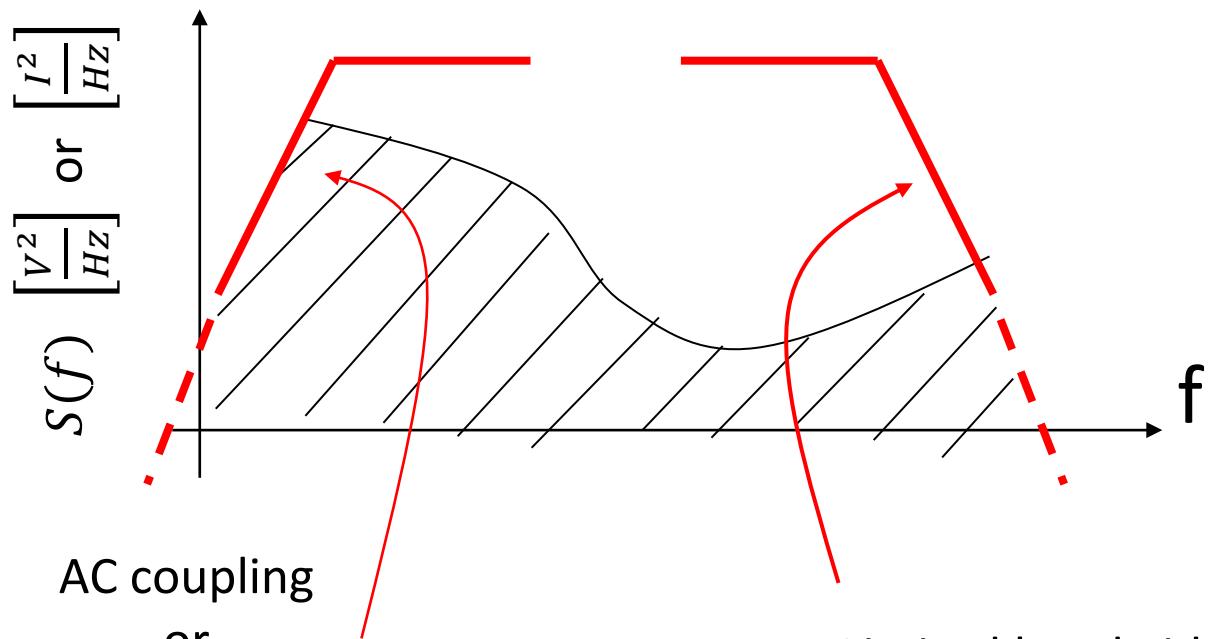


Drawings not to scale

# NOISE CHARACTERISTICS

## - *Limits in frequency* -

*Noise is not  $\infty$  (bandwidth is always limited) thanks to :*



AC coupling  
or  
limited time of the measurement  
or  
instrument reset

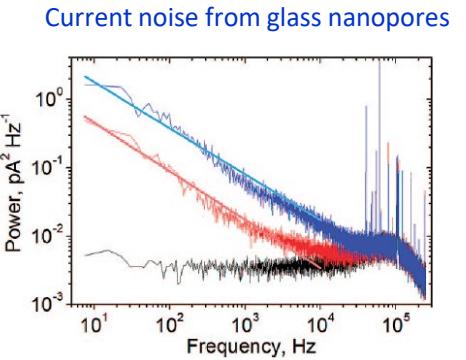
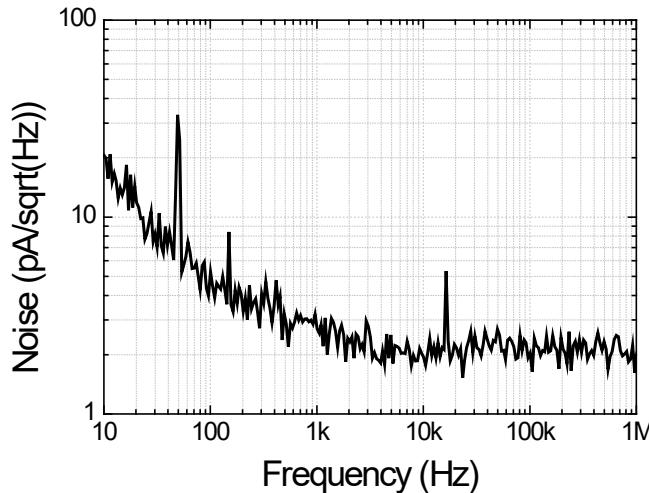
Limited bandwidth of the  
circuit/instrument

# LOW FREQUENCY NOISE

Electronic noise increases at low frequency (traps, mobility fluctuations, ...)

Mechanical vibrations  
Thermal fluctuations

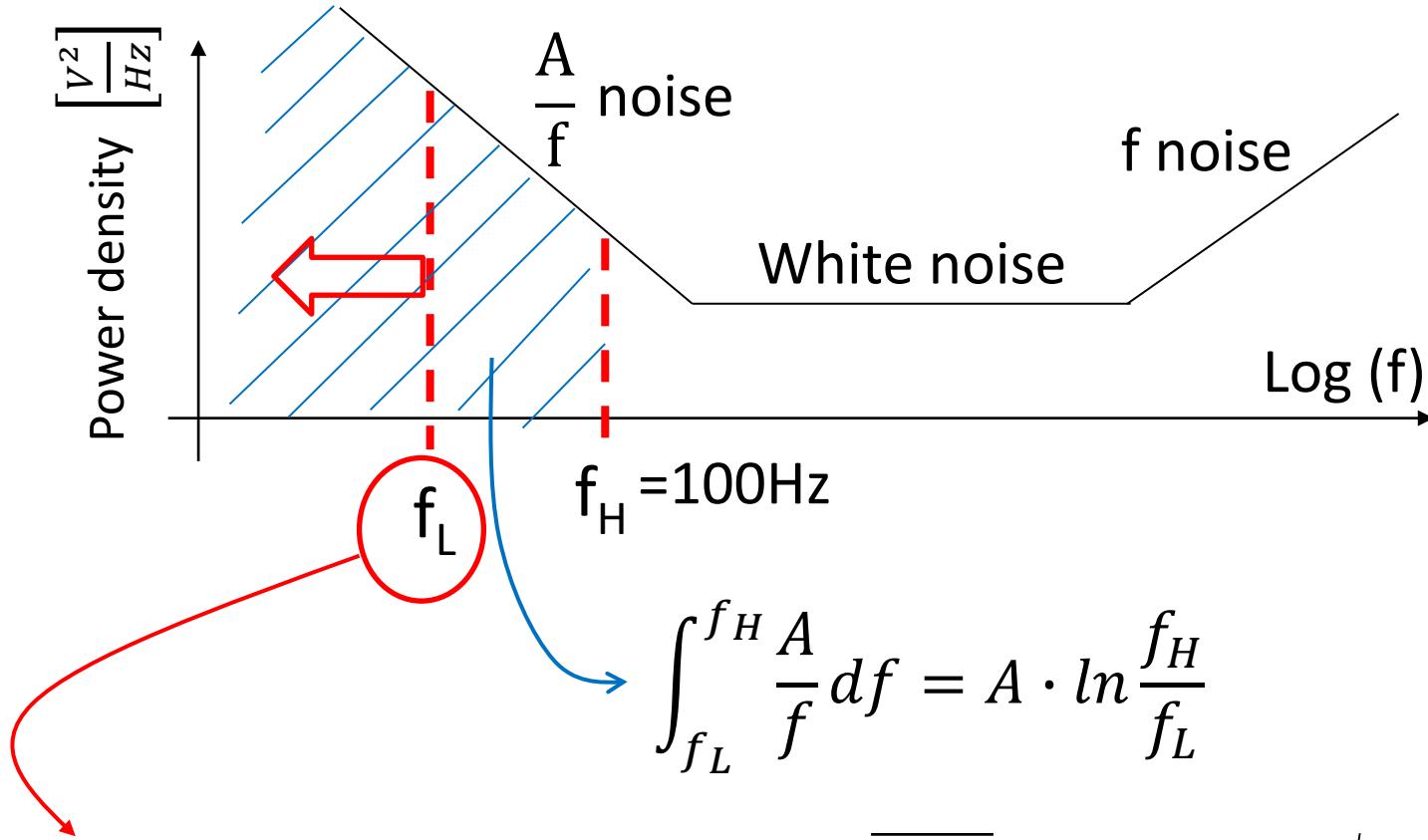
Bio-physical instability



Lesson on Friday

*Because of  $1/f$ , area of spectrum increases  $\Rightarrow$  more noise !*

# RMS of 1/f NOISE power spectrum



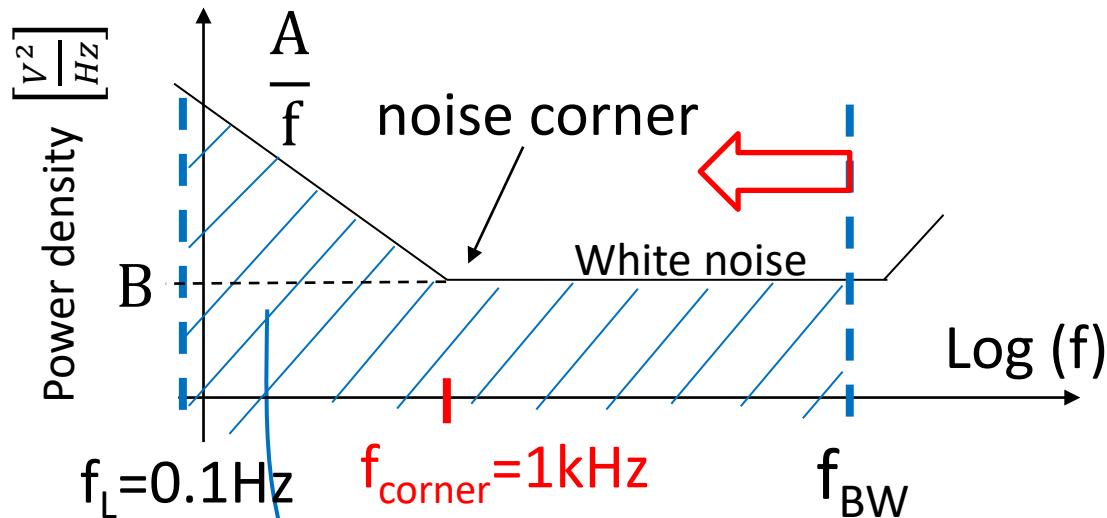
$$1 \text{ minute (60s)} \quad f_L \approx 1/60 \text{ Hz} \quad \rightarrow \quad \overline{n^2(t)} = A \cdot 9 \quad \rightarrow \text{rms} = \sqrt{A \cdot 3}$$

$$1 \text{ hour (3600s)} \quad f_L \approx 1/3600 \text{ Hz} \quad \rightarrow \quad \overline{n^2(t)} = A \cdot 13 \quad \rightarrow \text{rms} = \sqrt{A \cdot 3.6}$$

$$1 \text{ day (86400s)} \quad f_L \approx 1/86400 \text{ Hz} \quad \rightarrow \quad \overline{n^2(t)} = A \cdot 16 \quad \rightarrow \text{rms} = \sqrt{A \cdot 4}$$

*Noise does not increase much in extending measurement time*

# Noise corner frequency and RMS



$$A = 100 \cdot 10^{-15} [\text{V}^2]$$

$$B = (10 \text{nV}/\sqrt{\text{Hz}})^2$$

$$\int_{f_L}^{f_H} S(f) df \cong A \cdot \ln \frac{f_c}{f_L} + B \cdot f_{BW}$$

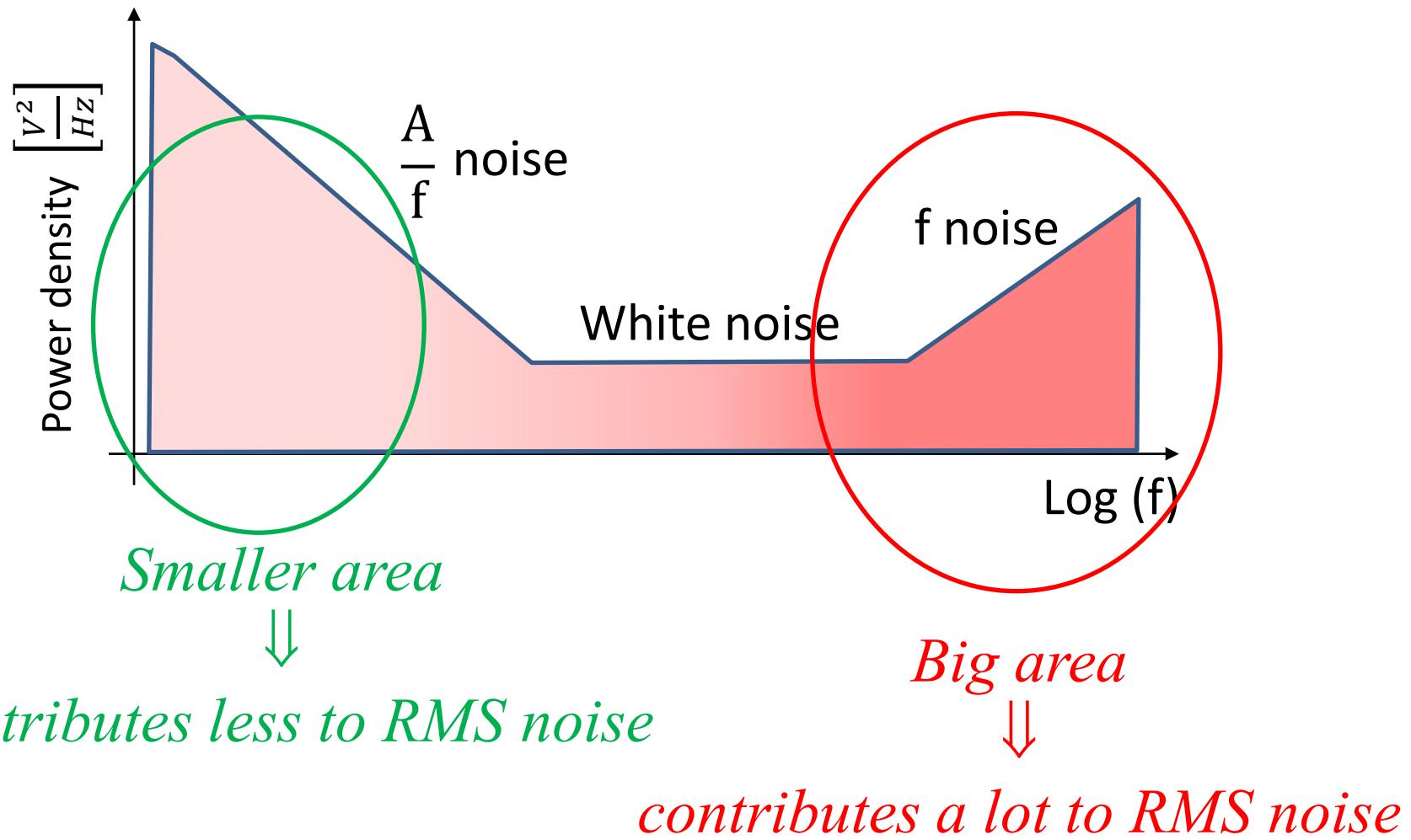
$$f_{BW} = 1 \text{MHz} \rightarrow \overline{n^2(t)} = 0.9 \cdot 10^{-12} + 100 \cdot 10^{-12} [\text{V}^2] \rightarrow \text{rms} = 10 \mu\text{V}$$

$$f_{BW} = f_c = 1 \text{kHz} \rightarrow \overline{n^2(t)} = 0.9 \cdot 10^{-12} + 0.1 \cdot 10^{-12} [\text{V}^2] \rightarrow \text{rms} = 1 \mu\text{V}$$

$$f_{BW} = 10 \text{ Hz} \rightarrow \overline{n^2(t)} = 0.5 \cdot 10^{-12} + 0.001 \cdot 10^{-12} [\text{V}^2] \rightarrow \text{rms} = 0.7 \mu\text{V}$$

*Longer averaging below f<sub>corner</sub> does not reduce RMS noise*

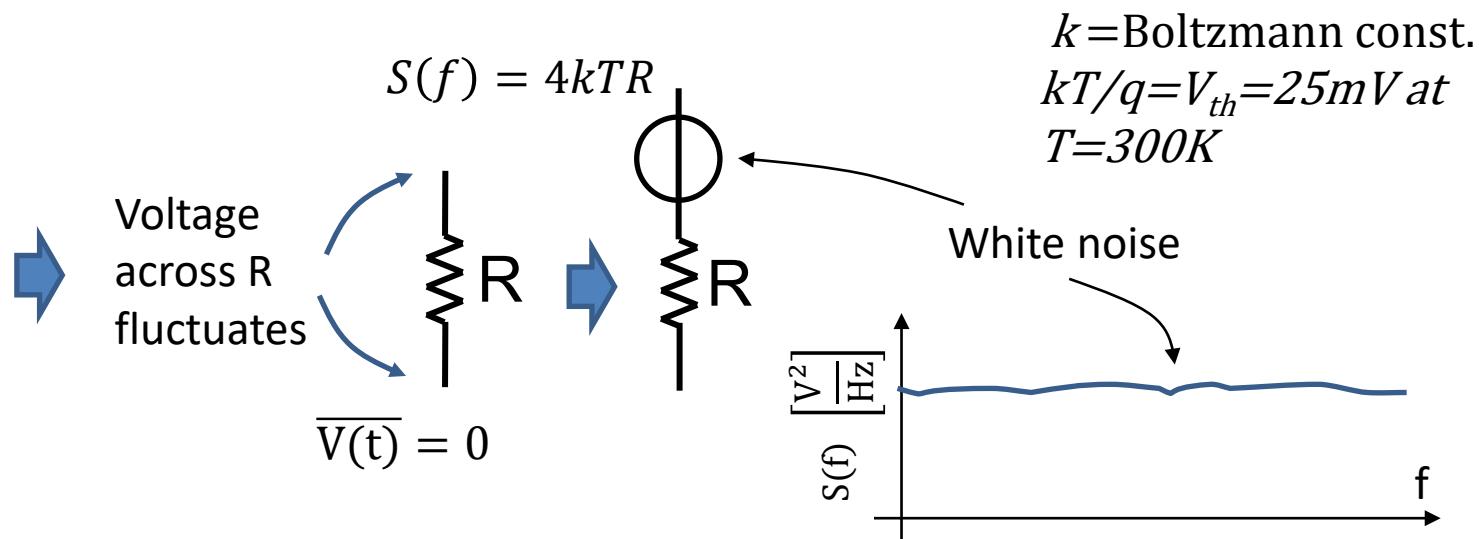
# Summary



*Do not forget that horizontal axis is logarithmic !*

# SOURCES of NOISE : RESISTORS

Thermally activated Brownian motion of the carriers



White noise value is independent of the amount of current flowing

**1/f noise** The case of a homogeneous resistor:

$$S_i(f) \approx \frac{\alpha}{Nf} I^2$$

*Hooge's empirical relation*

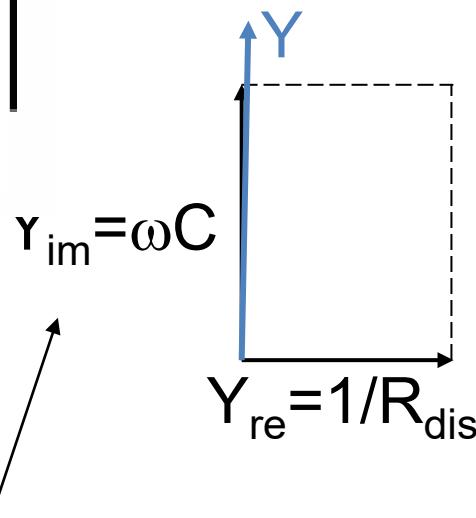
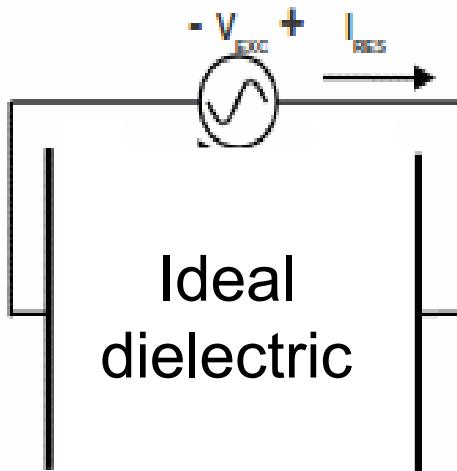
N = number of free charge carriers (*not concentration*)

$\alpha$  = Hooge parameter

$$\text{Noise} = \overline{i_{n,amp}^2} + \frac{4kT}{R_{\text{sensor}}} + \frac{\alpha}{Nf} \left( \frac{V_{\text{bias}}}{R_{\text{sensor}}} \right)^2$$

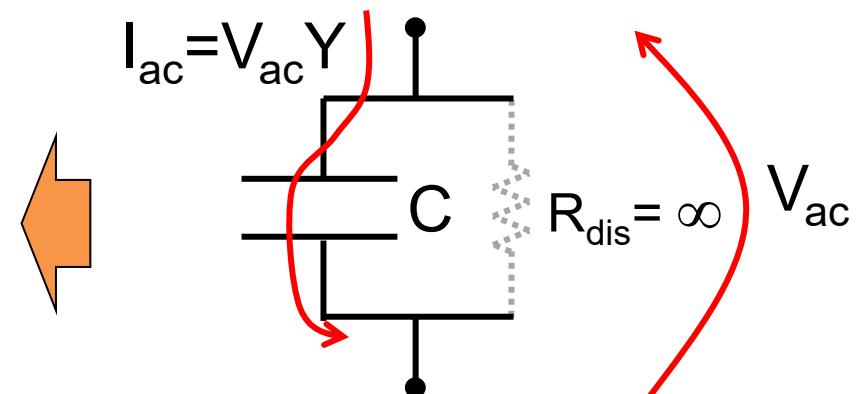
# SOURCES of NOISE : CAPACITORS

*Capacitors are considered noiseless*



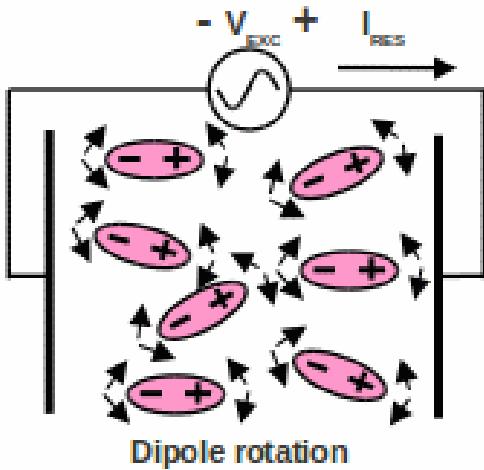
*Admittance  $Y_{\text{im}}$  proportional to frequency*

A capacitor with an ideal dielectric (air) has an ideal equivalent circuit.



# Real capacitor : dielectric noise

<http://www.lambient.com/>

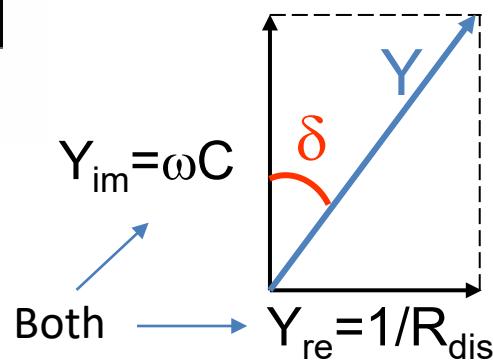
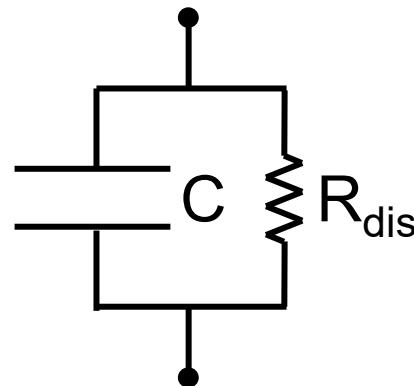
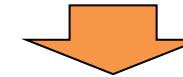


Dipole rotation

AC signal forces a dipole rotation



energy dissipation



Both

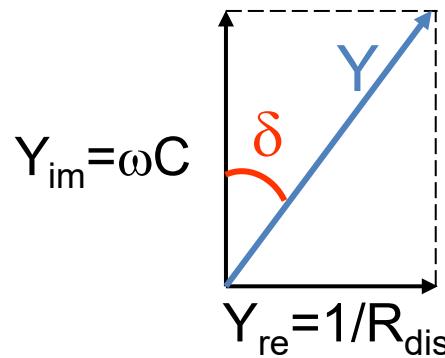
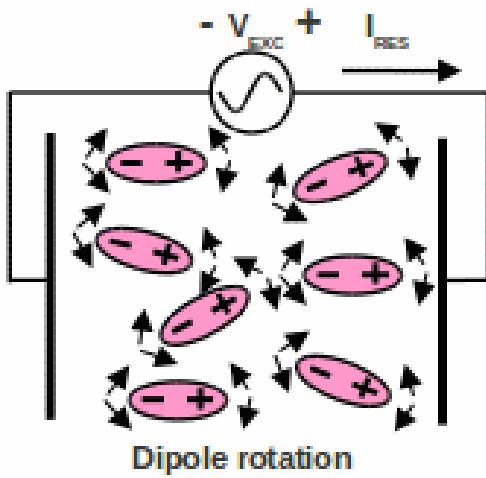
should stretch the same way !

$$\tan(\delta) = \frac{Y_{re}}{Y_{im}} = \frac{1}{\omega R_{dis} C} \approx const$$

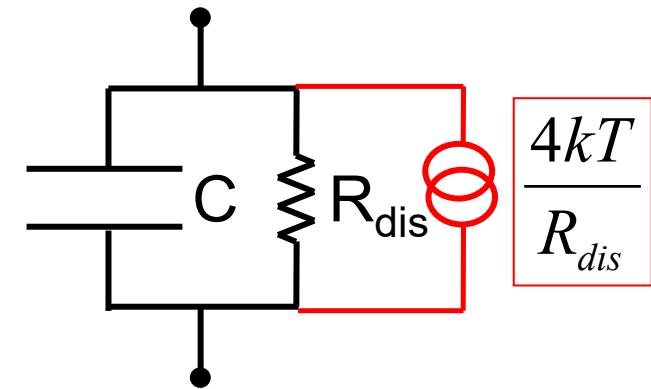
dissipation proportional to frequency

# Real capacitor : dielectric noise

An AC signal implies  
a dipole rotation



energy dissipation



$$\tan(\delta) = \frac{Y_{re}}{Y_{im}} = \frac{1}{\omega R_{dis} C} \approx const$$

often called “Dissipation factor”

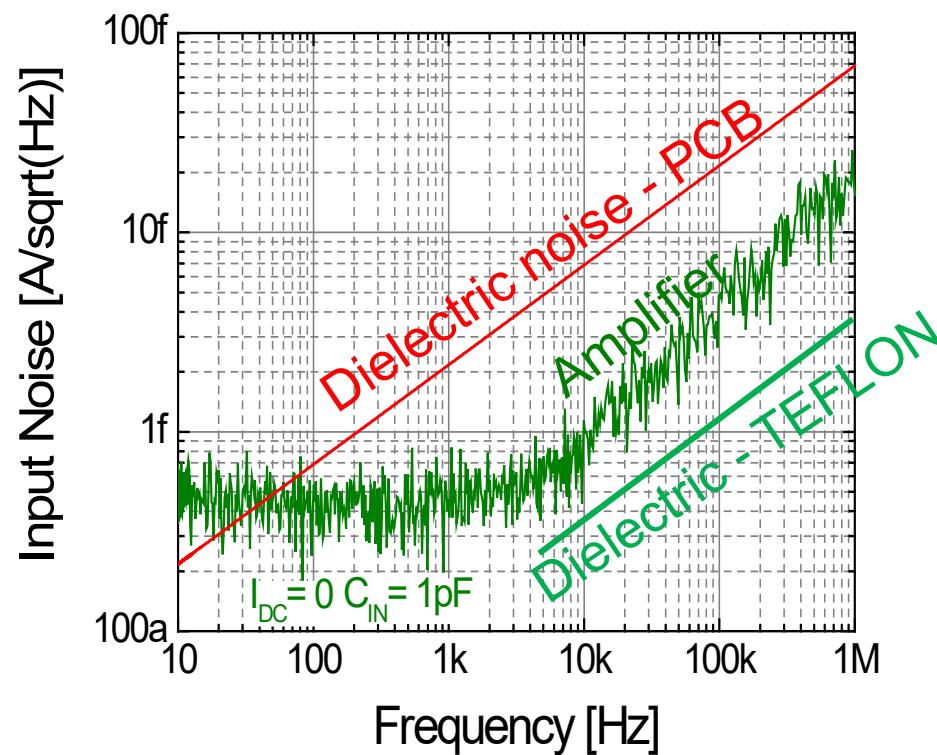
$$S_i = \frac{4kT}{R_{dis}} \approx 4kTC \tan(\delta) \omega$$

Increases with frequency !

# Dielectric noise

Dissipation factor,  $\tan(\delta)$ : PCB (FR4):  $2 \cdot 10^{-2}$   
Ceramic :  $10^{-3}$   
Teflon:  $10^{-4} - 10^{-5}$

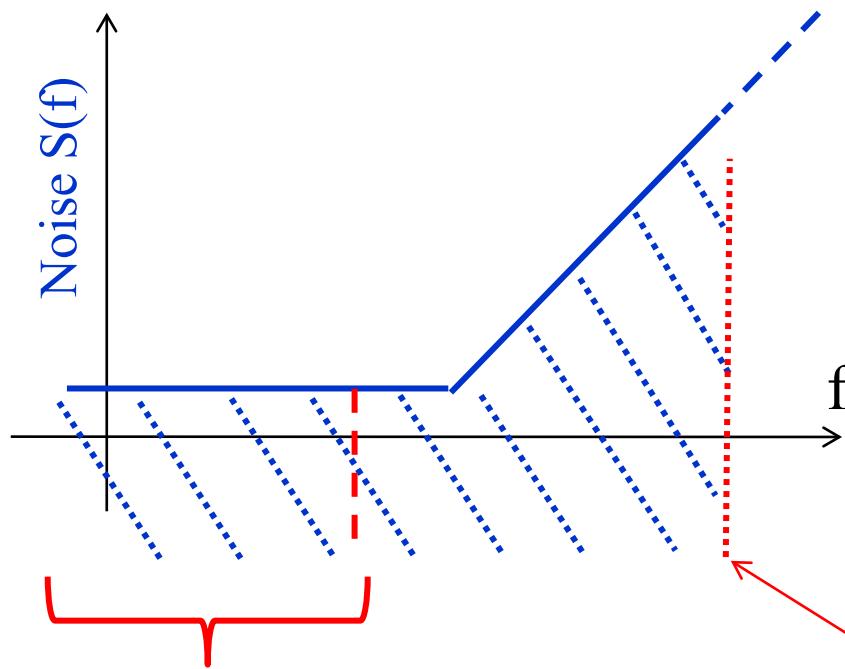
**Example** : 1cm x 1cm of PCB  $\rightarrow C = 2.6\text{pF}$





# In conclusion ...

# THINGS to REMEMBER (1)



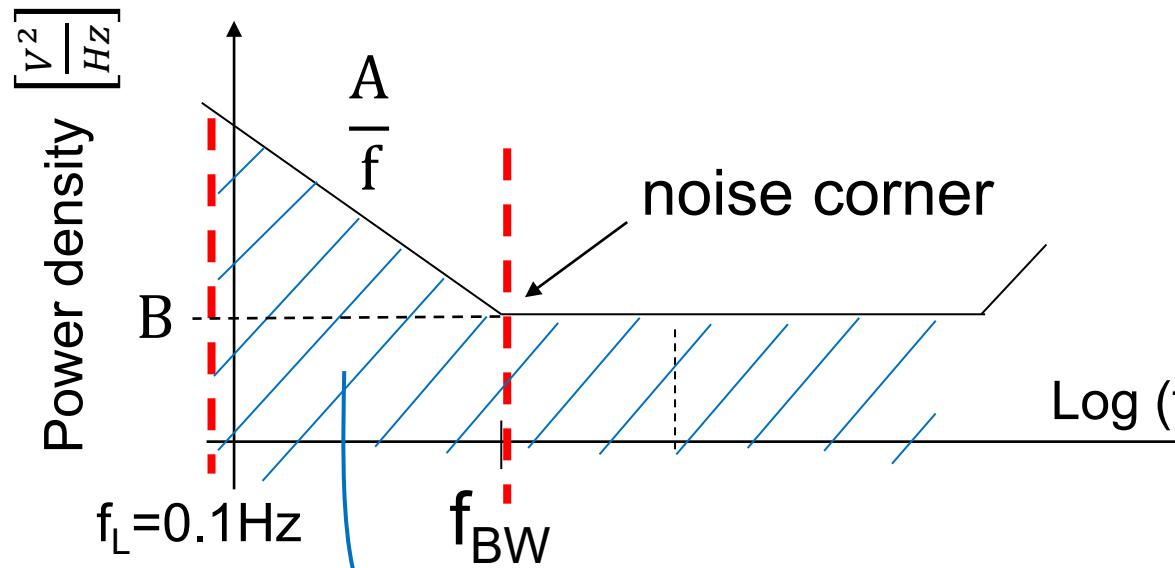
**Signal bandwidth**

(Physical limits of the sensor)

**Amplifier bandwidth**  
(sets the noise level)

**NEVER USE MORE BANDWIDTH THAN REQUIRED by the SIGNAL**

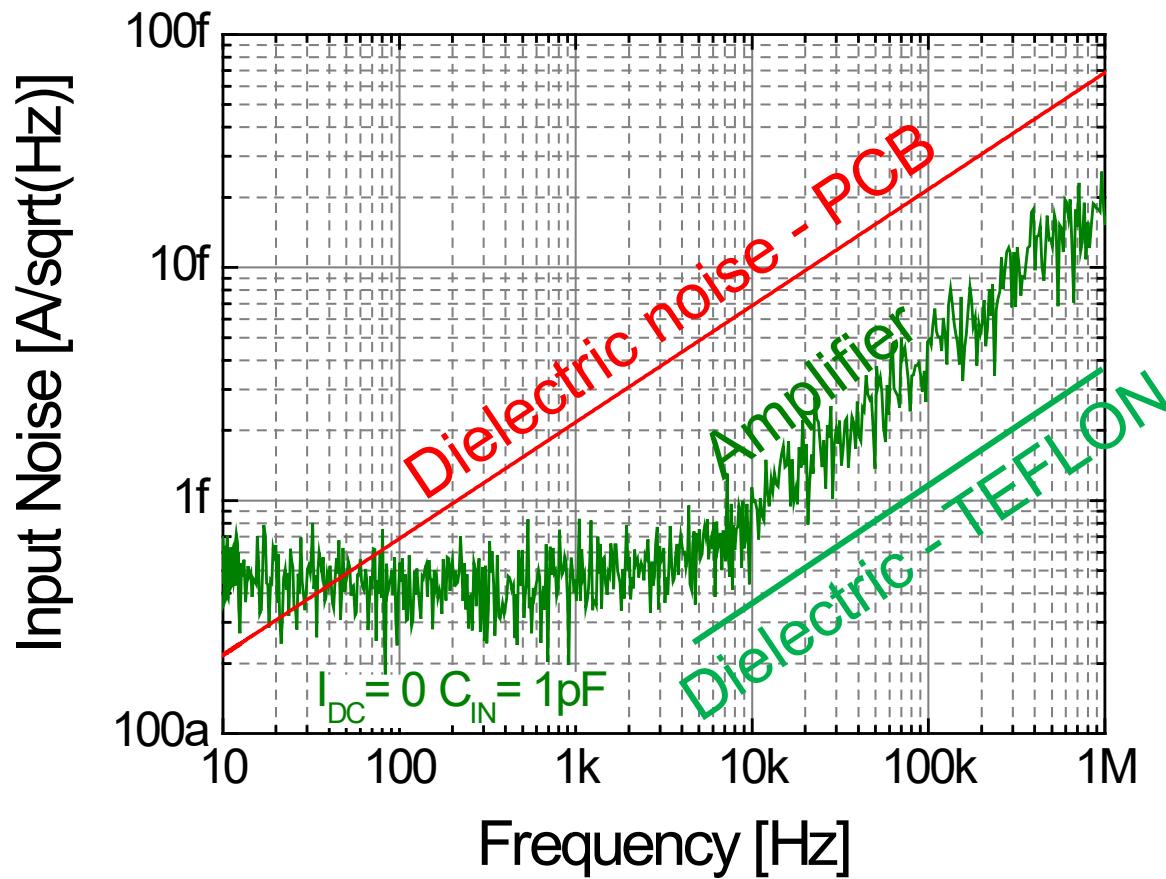
# THINGS to REMEMBER (2)



$$\int_{f_L}^{f_H} S(f) df \cong A \cdot \ln \frac{f_c}{f_L} + B \cdot f_{BW}$$

**NO ADVANTAGE IN GOING BELOW THE CORNER FREQUENCY**

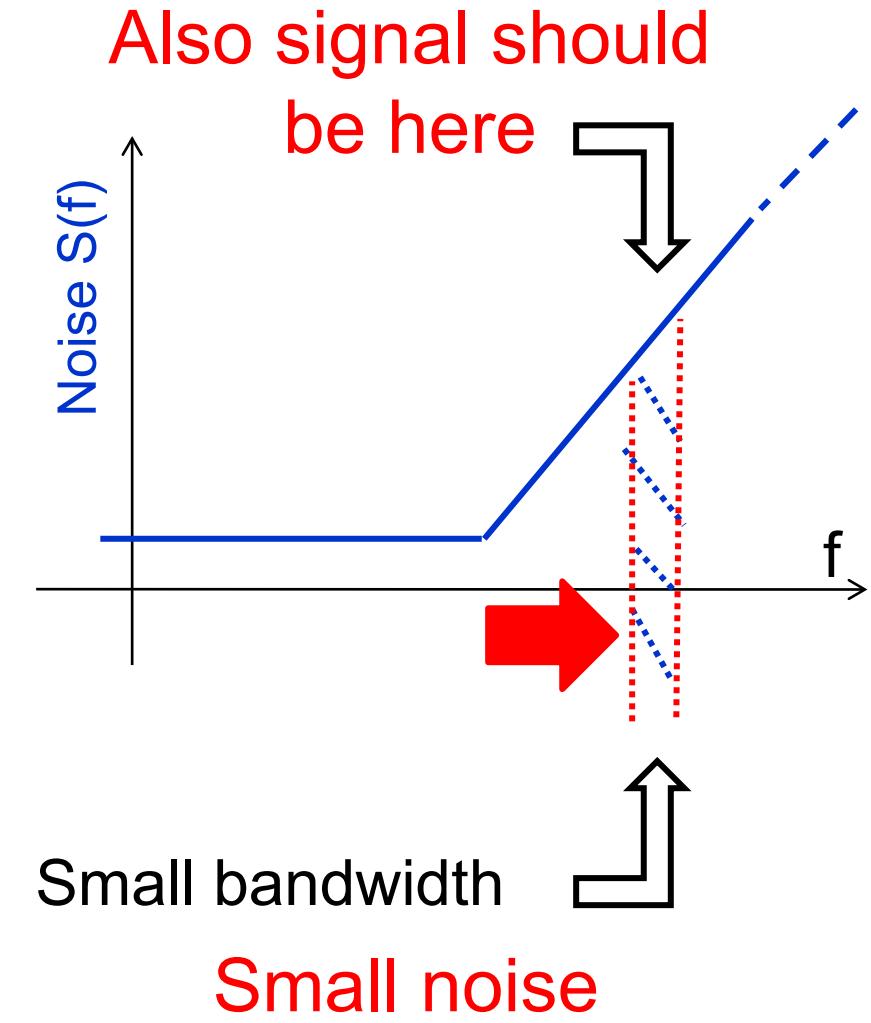
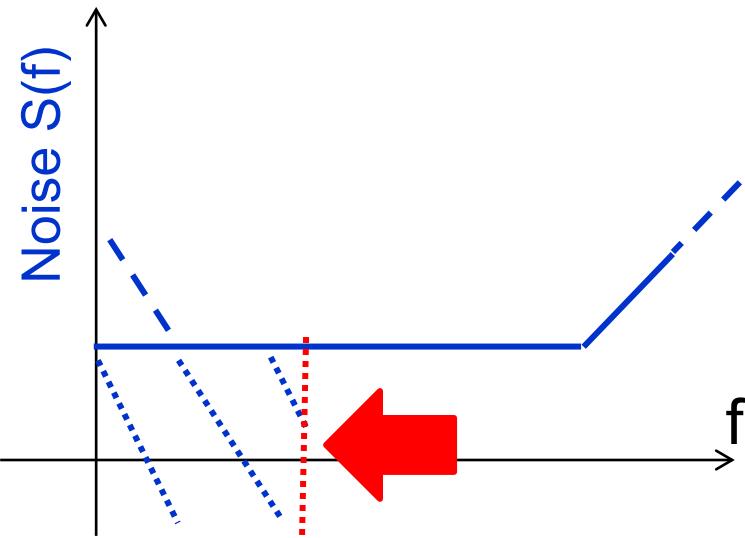
# THINGS to REMEMBER (3)



MATERIALS PLAY A ROLE IN ULTIMATE PERFORMANCE

What is next ?

... small bandwidth at high frequency



Small bandwidth

Small noise